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ABSTRACT

The concept of function is central in mathematics and grows in importance as one progresses in depth and breadth of understanding mathematics. Results from national assessment progress reports indicate that the majority of students do not comprehend the concepts of function and graphing. Technology has the potential to improve the understanding of functions and graphs. With the use of a Calculator-Based Laboratory system (CBL) and a graphing calculator, students are able to physically observe a functional relationship between two variables and observe a graphical representation of that relationship as a scatterplot. Several CBL mathematics experiments are presented as examples of the improvements in concept development possible with the use of calculator-based graphing. Contains 23 references. (ASK)

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Bring Functions and Graphs to Life with the CBL

A Presentation at the 1997 Carolinas Mathematics Conference
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Bring Functions and Graphs to Life with the CBL

Introduction

It can be argued that the function concept is the single most important concept from kindergarten to graduate school (Harel and Dubinsky, 1992). The function concept is a central one in mathematics which grows in importance as one progresses in the depth and breadth of one's understanding of mathematics (Yerushalmy and Schwartz, 1993). Yerushalmy and Schwartz believe that the function is the fundamental object of algebra. The American Mathematical Association of Two-Year Colleges [AMATYC] (1995) in its Standards document includes function as one of its Standards of Content and states, "Students will demonstrate understanding of the concept of function by several means (verbally, numerically, and symbolically) and incorporate it as a central theme into their use of mathematics."

Introduction of Functions and Graphs

The introduction of functions and graphs is a critical moment in mathematics education since it presents a setting with the opportunity for powerful learning to take place and since the concepts of functions and graphs are fundamental to more sophisticated parts of mathematics (Leinhardt, Zaslavsky, and Stein, 1990). Functions and graphs represent one of the earliest points in mathematics at which students use one symbolic system to expand and understand another. Algebraic and graphical representations are used to jointly construct and define the mathematical concept of function; consequently, functions and graphs cannot be treated as isolated concepts. Since functions and graphs are two symbolic systems used to illustrate each other, demands are placed on the learner in terms of new ideas, notational uniqueness, and symbolic correspondences.

Students' Difficulties with Functions and Graphs

Students experience a major leap in their mathematical development when they are introduced to the concept of the graph of a function in two variables (Herscovics, 1989). Results from the Second Mathematics Assessment of the National Assessment of Educational Progress (Carpenter, Corbitt, Kepner, Linquist, and Reys, 1981) indicate that the majority of students do not manage this leap. Carpenter et al. (1981) found that students could graph ordered pairs of numbers in the Cartesian plane but that most did not understand the relationship between equations and their graphs.

Results from the Fourth National Assessment of Educational Progress (Brown et al., 1988; Silver et al., 1988) indicated that U.S. students had a limited understanding of function concepts and of graphing. Brown et al, (1988) reported that secondary students demonstrated some intuitive knowledge of functions but experienced difficulty with items covering a broad range of function concepts.

Kenelly (1986) contends that calculus students experience difficulty with function concepts. He surmises that beginning algebra students fail to form a conceptual understanding of variables; as a result, for many students variables are simply symbols used in manipulative practice exercises and functions are "ordered pairs of these things." Students miss the idea that functions capture the spirit and essence of connections and interdependencies, and they fail to see that functions embrace the elements of input and output, control and observation, and cause and effect. Epps (1986) says that beginning calculus students do not know the abstract definition of graph of a function. They can plot and connect points, but they do not know that for a function f, f(x) is the height to the graph of f at x.

The Potential of Technology

Technology has the potential to enhance the understanding of functions and graphs, but this potential has not been fully realized. The hand-held scientific calculator was introduced in the 1970s, but McConnell (1988) states that even though calculators are found in every nook and cranny of the United States, many mathematics students are still subjected to topics that assume a world with no calculators and are forbidden to use calculators in their classrooms. The microcomputer has not had the impact on the teaching and learning of mathematics that had been predicted (Barrett and Goebel, 1990) since many schools do not have a computer in each mathematics classroom and since many educators have trouble defining the role of the computer in the classroom.

Most algebra teachers indicate that they use computers for demonstration purposes only (Demana and Waits, 1992). The computer has had a great impact on mathematics, but mathematics is still being taught in most college courses just as it was 30 years ago as a paper-and-pencil discipline (National Research Council, 1991). Demana and Waits (1992) are convinced that if teachers and students rely solely on desktop personal computers, no meaningful reform will occur in mathematics education in the 1990s. Demana and Waits advocate the use of inexpensive pocket computers (graphing calculators) in mathematics education.

Computer- and calculator-based graphing has the potential to change the way mathematics is taught and learned (Waits and Demana, 1988) since this technology enables students to handle more complicated, realistic, and noncontrived applications. The graphing calculator can transform the mathematics classroom into a laboratory environment where students use technology to investigate, conjecture, and verify their findings (National Council of Teachers of Mathematics, 1989).

Graphing calculators have the greatest potential of all of technological innovations to impact on the teaching of precalculus and calculus (Dion, 1990).

Effect of Technology on Mathematics Education

Several studies have yielded inconclusive evidence of the value of the graphing calculator in mathematics education. Rich (1990/1991) conducted a study to examine the effects of graphing calculators on precalculus students' achievements, attitudes, and problem-solving approaches in which two classes were taught precalculus using CASIO fx-7000G graphing calculators and six classes were taught precalculus without graphing calculators. The study did not provide evidence of an overall achievement effect of graphing calculator instruction. Rich concluded that students taught precalculus with the use of graphing calculators acquire a better understanding of the relationship between an algebraic equation and its graph.

Estes (1990) conducted a study to investigate the effects of implementing graphing calculators and computer technologies as teaching tools in Applied Calculus in which she designed and taught two experimental classes in Applied Calculus with the use of CASIO fx-7000G graphing calculators and a Macintosh SE computer with overhead projector capabilities. Special assignments were developed and used to facilitate learning the concepts of Applied Calculus with a graphing calculator. Students in a control group were allowed to use calculators, but did not receive any special attention to effective uses of the graphing calculator to facilitate conceptual learning. On end-of-semester tests, the experimental group scored significantly higher than the control group on conceptual measures. No significant difference was found between the groups on procedural measures.

Caldwell (1994/1995) conducted a study to determine the effect of the TI-81 Graphics Calculator as a learning tool on college students' understanding of concepts and performance of procedures involving functions and graphs. Two college algebra classes, the treatment group, studied functions and graphs with the use of TI-81 Graphics Calculators. Two college algebra classes, the control group, studied functions and graphs with the use of scientific calculators without graphing capabilities. At the conclusion of units on functions and graphs, a concepts test and a procedures test were administered to both treatment and control groups. The treatment group scored significantly higher than the control group on the procedures test. There was no significant difference between the two groups on the concepts test.

Graphing calculators provide opportunities for students to connect graphical images, symbolic expressions, and sets of related numerical values (Caldwell, 1994/1995). With graphing calculators such as the TI-82 and TI-83, students can represent functions numerically as tables of input-output pairs, symbolically as algebraic representations, and graphically as plots of input-output points. Students need to translate across these representations and connect these representations to physical contexts. A device which can aid students in connecting numerical, symbolic, and graphical representations of functions to physical contexts is the Texas Instruments Calculator-Based Laboratory System (CBL).

The Calculator-Based Laboratory System

With the use of a CBL and a graphing calculator, students are able to physically observe a functional relationship between two variables and then observe a graphical representation of the relationship as a scatter plot. Students may also observe a numerical representation of the relationship by tracing among the points of the scatter plot and by scanning the lists in which data

are stored. Students may then use the graphing calculator to determine an equation for a curve to be fitted to the data and thus obtain a symbolic representation of the relationship.

The CBL is a portable, handheld, battery-operated data collection device for collecting real-world data (Texas Instruments Incorporated, 1994a). With the CBL and appropriate probes or sensors, one can measure motion, force, temperature, light, sound, pressure, and more. Data can then be transferred directly to TI-82, TI-83, TI-85, and TI-92 graphing calculators for analysis. The data is stored in lists in the graphing calculator, and data points are plotted as a scatter plot on the calculator screen. Several books have been published with step-by-step mathematics and science experiments for the CBL including programs for the experiments that can be downloaded into graphing calculators. Some of the experiments which have been used by the author are described in this paper.

CBL Mathematics Experiments

Using the HIKER program (Texas Instruments Incorporated, 1994a) or the VELOCITY program (Meridian Creative Group,1996) with a CBL, a graphing calculator, and a motion detector, students walk in such a way that the graphs of their distances from the motion detector with respect to time form different shapes. Experiments with the HIKER or VELOCITY programs allow students to develop the concept of slope. Walking away from the motion detector at a constant rate generates a line with a positive slope as illustrated in Figure 1while walking toward the motion detector at a constant rate generates a line with a negative slope. By varying one's rate during a single walk or by walking away from and toward the motion detector during a single walk one may generate graphs of many different shapes as shown in Figure 2. These graphs represent functions, and by tracing among the points on the graphs, one

may determine domains, ranges, and extrema of functions. Experiments using the HIKER or VELOCITY programs provide opportunities for students to develop and refine such concepts as function as a correspondence; function as a set of ordered pairs; domain and range of a function; relative maxima and minima; absolute maxima and minima; and increasing, decreasing, and constant functions.

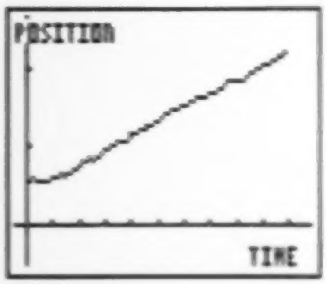

Figure 1. Graph generated by walking away from the motion detector at a constant rate.

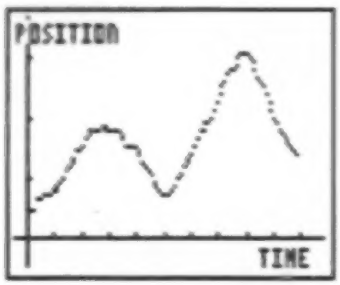

Figure 2. Graph generated by walking away from and toward the motion detector at varying rates.

The SLOPE program (Meridian Creative Group, 1996) may be used to introduce the concept of slope as a rate. Using a CBL, a motion detector, and a graphing calculator, a student walks away from the motion detector two times at two different speeds. Equations representing the speeds in meters per second are plotted on the calculator screen as shown in Figure 3. Students can analyze the graphs and determine the walker's speeds. Students can be asked to draw conclusions about the relationship between the slope of a line and the speed of a walker. Students can also be given a screen output such as the one in Figure 4 and asked to describe how to repeat the experiment in order to obtain similar graphs.

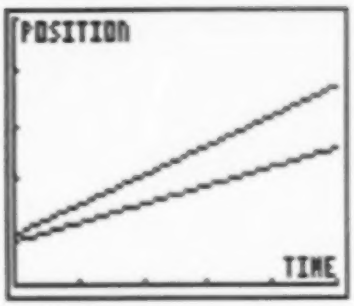

Figure 3. Graph representing a walker's two speeds walking away from the motion detector.

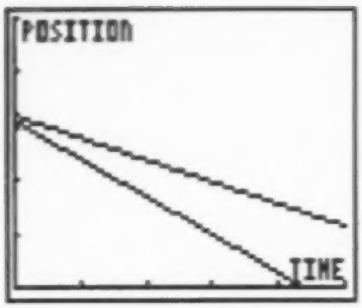

Figure 4. Graph representing a walker's two speeds walking toward the motion detector.

Several CBL programs allow students to work with specific types of functions. The PENNIES program (Texas Instruments Incorporated, 1994b) presents an application of a linear function. Using the PENNIES program, a CBL, a TI-82, and a force sensor, students drop four sets of eight pennies into a cup suspended from the force sensor. The numbers of pennies and the masses of the pennies are recorded, and a scatter plot is plotted on the calculator screen as illustrated in Figure 5. Students can trace among the points of the scatter plot, analyze the data, determine the slope of a line passing through two of the points, attempt to interpret the meaning of slope as it relates to the number of pennies placed into the cup and the mass of the pennies, and determine an equation of a line passing through two of the points. By selecting LinReg from the STAT menu on the calculator, the student can find an equation for a line of best fit, paste this equation into the Y= list and then plot the line on the calculator screen along with the scatter plot of the collected data.

The LEVER program (Meridian Creative Group, 1996) illustrates another application of linear functions. Using the LEVER program, a CBL, a force sensor, a ruler, and a small mass, students place the small mass at several different distances from a fulcrum. The distances and forces are

recorded, and a scatter plot of force in Newtons with respect to distance in centimeters is plotted on the calculator screen as shown in Figure 7. Students can analyze this data as they would analyze the data from the PENNIES experiment.

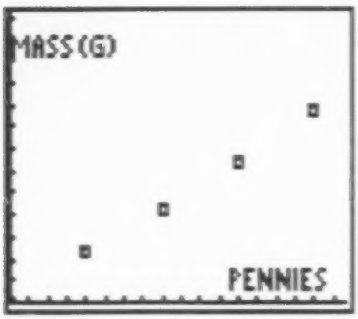

Figure 6. Graph showing the relationship between number of pennies and mass.

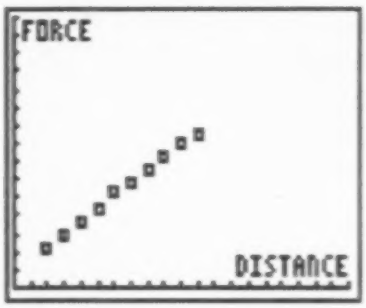

Figure 7. Graph showing the relationship between force and distance.

Using the VERTICAL program (Meridian Creative Group, 1996), a CBL, a graphing calculator, and a motion detector, one tosses an object straight up directly over the motion detector. The CBL collects data for 1.6 seconds, the data is transferred to the graphing calculator, and a scatter plot of height with respect to time is displayed. A prompt is displayed for one to choose a lower bound and an upper bound to eliminate points on the scatter plot representing heights and times before the object is released and after the object is caught. The plot of the remaining data will appear to be quadratic as illustrated in Figure 8. By tracing among the points of the final display, a student may approximate the velocity of the object at different times, and this can lead to a discussion of limits. The student can select QuadReg from the STAT menu and find an equation of a quadratic function to model the relationship between height and distance.

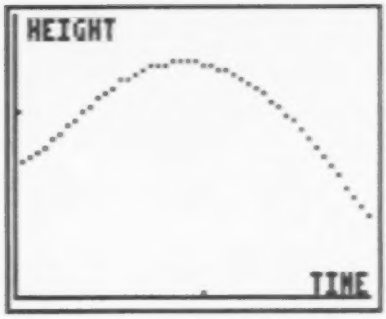

Figure 8. Graph showing the relationship between height and time.

The FORCE1 program used with a CBL, a graphing calculator, a force sensor, and a slinky may provide an illustration of harmonic motion. The slinky is suspended from the force sensor and then set in motion and the FORCE1 program is started. The CBL collects data and a scatter plot of force with respect to time is plotted as illustrated in Figure 9. The plot of the resulting data can be used to illustrate the concepts of periodic function, amplitude, period, phase shift, and vertical translation. By tracing among the points on the scatter plot, the student may find values of A, B, C, and D to fit an equation of the form $y = A\cos(Bx + C) + D$ to the scatter plot. The student can select SinReg from the STAT menu on a TI-83 and find an equation of a sine curve to fit the scatter plot. The SOUND program (Meridian Creative Group, 1996) can be used with a CBL, a graphing calculator, a microphone, and a tuning fork to illustrate periodic functions as illustrated in Figure 10.

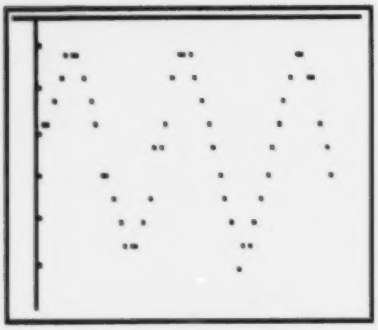

Figure 9. Graph showing the relationship between force and time.

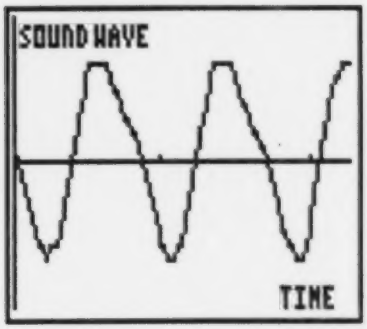

Figure 10. Graph showing the relationship between sound intensity and time.

The LIGHT program (Texas Instruments Incorporated, 1994a) or the INTENSITY program (Meridian Creative Group, 1996) may be used to illustrate power functions or the inverse square law. The LIGHT program requires a CBL, a graphing calculator, a light sensor, a light source, and a meter stick. The light sensor is placed at several different distances from the light source, the intensity of the light at each distance is measured by the CBL, and the resulting scatter plot of intensity with respect to distance is plotted on the calculator screen as shown in Figure 11. By tracing among the points of the scatter plot a student may approximate rates of change of intensity with respect to distance. The student can select PwrReg from the STAT menu and find an equation of a power function to fit the plotted data.

The HEAT program (Texas Instruments Incorporated, 1994) or the COOLING program (Meridian Creative Group, 1996) can be used with a CBL, a graphing calculator, a temperature probe, and different pieces of apparatus in the study of exponential functions. With the HEAT program or the COOLING program, the CBL collects several readings of time and temperature.

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After the data are collected, a scatter plot similar to the one shown in Figure 12 is displayed on the calculator screen.

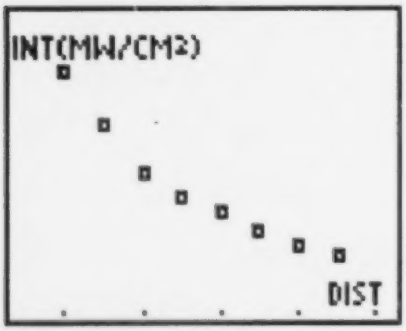

Figure 11. Graph showing the relationship between light intensity and distance.

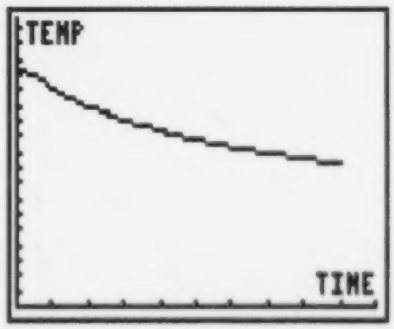

Figure 12. Graph showing the relationship between temperature and time.

An experiment can be conducted by heating a cup of water, removing the heat source, placing the temperature probe in the water, and then letting the CBL collect data over a period of several minutes. One may trace among the data points and approximate the rate of change of temperature at different times. By selecting ExpReg from the STAT menu on the calculator, the student may find an equation of an exponential function of the form $y = ab^x$ to model the relationship between time and temperature of a cooling object. By using natural logarithms, one can determine an equation of a corresponding exponential function with base e. The equation of the exponential function can be pasted into the Y= list and plotted on the calculator screen along with the scatter plot of the data.

An experiment requiring less time can be conducted by folding a piece of aluminum foil, placing the temperature probe between the layers of the foil, flattening the foil tightly around the probe, and then heating the foil with a hair dryer. When the foil is hot, the hair dryer is turned off

and the COOLING or HEAT program is started. Data points are collected by the CBL and plotted on the calculator screen as the foil cools. An exponential function can then be determined to model the data. An experiment requiring even less time can be conducted by placing the temperature probe into a cup of hot water for a short period of time and then removing it and placing it into a cup of ice water, starting the data collection with the COOLING or HEAT program as the temperature probe is removed from the hot water. The temperature probe can also be placed into a cup of ice water and then removed and placed into a cup of hot water generating a heating curve. Be selecting LnReg from the STAT menu on the calculator, one can find a logarithmic function to model the data.

Conclusions

One of the basic principles of the AMATYC Standards is that mathematics must be taught as a laboratory discipline (AMATYC, 1995). There must be an emphasis on effective mathematics instruction involving active student participation and in-depth projects employing genuine data to promote students' learning through guided hands-on investigations. The use of the CBL along with the graphing calculator promotes this type of learning environment.

With only one CBL in the classroom, students can physically observe a functional relationship between two variables while data is being collected by the CBL. Data can then be transmitted from the graphing calculator used with the CBL to students' graphing calculators. Students can then analyze the numerical representation of the data by scanning the lists in which the data are stored. Students can plot the data on their calculator screens to obtain a graphical representation of the data. By analyzing the plotted data points, students can determine the type of function that the points seem to fit and then use the graphing calculator to create an equation for the curve of

best fit to obtain a symbolic representation of the data. Students can then plot the curve of best fit on their calculator screens along with the scatter plot; using the curve of best fit, they can predict values of one variable when given values of the other variable.

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